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Irrational Numbers

Unit 4 Lesson 6

Students will be able to:

Understand the meanings of
Irrational Numbers

Key Vocabulary:

- Irrational Numbers
- Examples of Rational Numbers and Irrational Numbers
- Decimal expansion of Irrational Numbers
- Steps for representing Irrational Numbers on number line

IRRATIONAL NUMBERS

A rational number is a number that can be expressed as a ratio or we can say that written as a fraction. Every whole number is a rational number, because any whole number can be written as a fraction. Numbers that are not rational are called irrational numbers. An Irrational Number is a real number that cannot be written as a simple fraction or we can say cannot be written as a ratio of two integers.

The set of real numbers consists of the union of the rational and irrational numbers.

If a whole number is not a perfect square, then its square root is irrational. For example, 2 is not a perfect square, and $\sqrt{2}$ is irrational.

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Examples of Rational Number

The number 7 is a rational number because it can be written as the fraction $\frac{7}{1}$.

The number 0.1111111....(1 is repeating) is also rational number because it can be written as fraction $\frac{1}{9}$.

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Examples of Irrational Numbers

The square root of 2 is an irrational number because it cannot be written as a fraction $\sqrt{2}$ = 1.4142135……

Pi(π) is also an irrational number. π = 3.14159265358979323846264338327950 (and more...)

The approx. value of $\frac{22}{7}$ = 3.1428571428571... is close but not accurate. Another hint is that without repeating decimal goes on forever.

EXAMPLES OF RATIONAL NUMBERS AND IRRATIONAL NUMBERS

Some more examples of irrational numbers are -

$\sqrt[3]{3}$ (the cube root of 3) and the natural logarithm base e. The quantities $\sqrt{2}$ and $\sqrt[3]{3}$ are examples of algebraic numbers. **Pi** (π) and **e** are examples of special irrationals called as transcendental numbers.

The decimal expansion of an irrational number is always nonrepeating (the digits display no repetitive pattern) and nonterminating (it never ends).

If x and z are irrationals such that $x < z$, then there always exists an irrational y such that $x < y < z$. The set of irrationals is "more dense" than set of rational numbers.

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For Decimal Expansion of Irrational numbers we will take an example

Find Decimal Expansion of $\sqrt{3}$

$\sqrt{3}$ is an irrational number, we will do its decimal expansion $\sqrt{3}$ is between the two perfect squares $\sqrt{1}$ and $\sqrt{4}$. So, $\sqrt{3}$ is between 1 and 2.

To get more precise we will look at the tenths between 1 and 2.

Is $\sqrt{3}$ is between 1.2 and 1.3?

Try and check $1.2^2 < 3 < 1.3^2$. But $1.2^2 = 1.44$ and $1.3^2 = 1.69$ these squares are too small.

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Is $\sqrt{3}$ is between 1.8 and 1.9?

Try and check $1.8^2 < 3 < 1.9^2$. But $1.8^2 = 3.24$ and $1.9^2 = 3.81$ these squares are too big.

Is $\sqrt{3}$ is between 1.7 and 1.8?

Try and check $1.7^2 < 3 < 1.8^2$; $1.7^2 = 2.89$ and $1.8^2 = 3.24$; $2.89 < 3 < 3.24$ therefore $1.7 < \sqrt{3} < 1.8$ so, $\sqrt{3}$ lies between 1.7 and 1.8.

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DECIMAL EXPANSION OF IRRATIONAL NUMBERS

For the next decimal look at the tenths between 1.7 and 1.8 by trial and error method we found that $\sqrt{3}$ lies between 1.73 and 1.74 as $1.73^2 = 2.9929$ and $1.74^2 = 3.0276$.

Therefore, first two decimal place values of $\sqrt{3}$ is 1.73….

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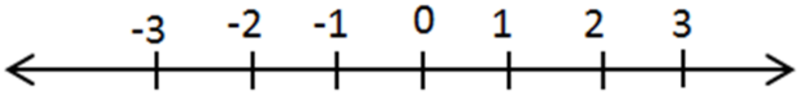
STEPS FOR REPRESENTING IRRATIONAL NUMBERS ON NUMBER LINE

For representing irrational number on number line we will take an example

Represent $\sqrt{2}$on the number line.

Step1: Draw a number line.



Step2: With same length as between 0 and 1, draw a line perpendicular to point (1), such that new line has a length of 1 unit.

Step3: Now join the point (0) and the end of new line of unity length.

Step4: A right angled triangle is constructed.

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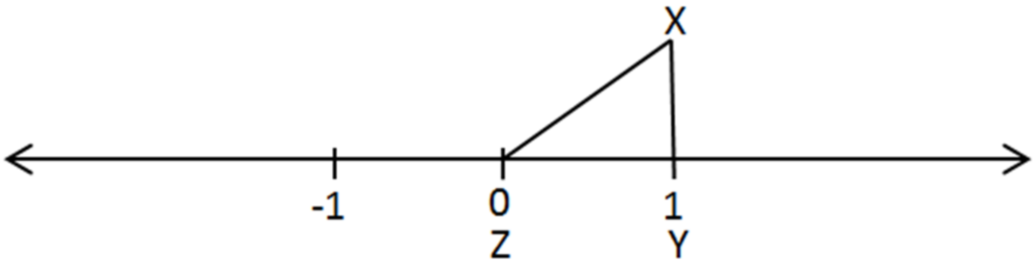
STEPS FOR REPRESENTING IRRATIONAL NUMBERS ON NUMBER LINE

Step5: Now let us name the triangle as XYZ such that XY is the height (perpendicular), YZ is the base of triangle and XZ is the hypotenuse of the right angled triangle XYZ.



Step6: Now length of hypotenuse, i.e., XZ can be found by applying Pythagoras theorem to the triangle

$(XZ)^2 = (XY)^2 + (YZ)^2$

$(XZ)^2 = (1)^2 + (1)^2$

$(XZ)^2 = 2 \rightarrow XZ = \sqrt{2}$

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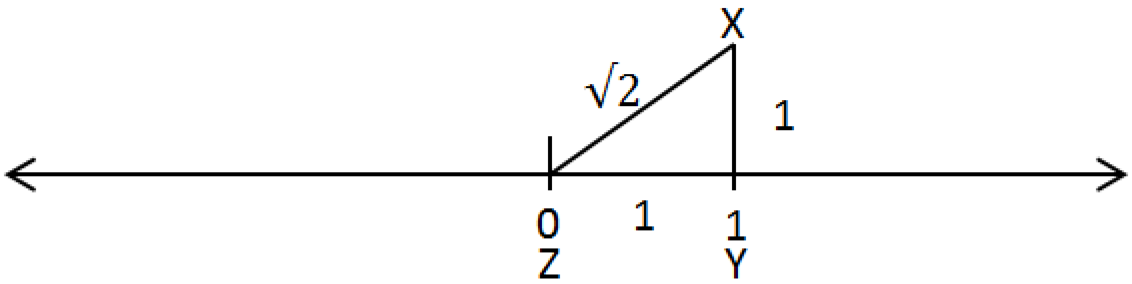
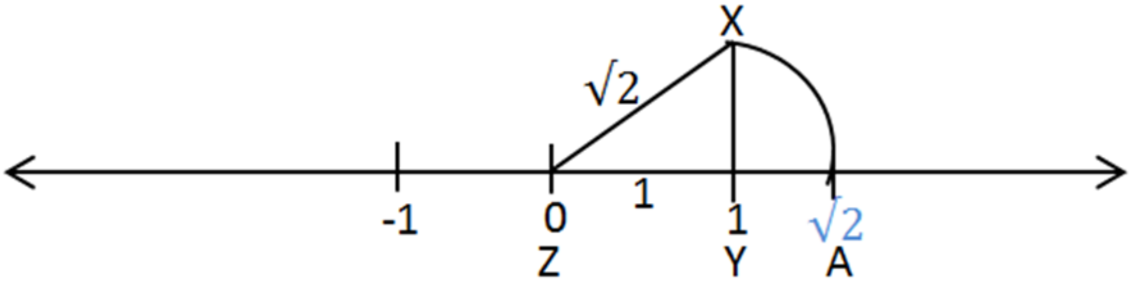
STEPS FOR REPRESENTING IRRATIONAL NUMBERS ON NUMBER LINE



Step7: Now with XZ as radius and Z as the center cut an arc on the same number line and name the point as A.

Step8: Since XZ is the radius of the arc and hence, ZA will also be the radius of the arc whose length is $\sqrt{2}$.

Step9: Hence, A is the representation of $\sqrt{2}$ on the number line.



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Exercise

Find whether the following numbers are rational or irrational.

(a) 0.23520678…..

(b) 0.33333333…..

ANSWERS

Find whether the following numbers are rational or irrational.

(a) 0.23520678….. **Irrational Number** (the digits display no repetitive pattern)

(b) 0.33333333….. **Rational Number** (the digits display repetitive pattern)