When we evaluate expressions, we use one set of rules so that everyone arrives at the same correct answer.

The rules used for simplifying numerical expressions are called **order of operations.**

These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally, the least powerful ones last (addition and subtraction, going from left to right).

Grouping symbols, like parentheses, tell us to evaluate whatever is inside them before moving on.

You can remember the order of operations with the acronym **PEMDAS**.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Please | Excuse | My | Dear | Aunt | Sally |
| Parentheses | Exponents | Multiplication | Division | Addition | Subtraction |
| **P** | **E** | **M** | **D** | **A** | **S** |

**Expressions with Only Addition, Subtraction, Multiplication, and Division**

Multiplication and division are evaluated first, from left to right.

Addition and subtraction are always evaluated last, from left to right.

**Sample Problem 1**: **Find the value of each numerical expression.** **Follow the order of operations when finding each value.**

|  |  |  |
| --- | --- | --- |
| **a.** | $$16-12÷2+4=$$ | $$ 16-12÷2+4=$$$$=16-6+4=$$$$=10+4=$$$$=16$$ |
| **b.** | $$196\*10-26=$$ | $$ 196\*10-26=$$$$=1,960-26=$$$$=1,934$$ |
| **c.** | $$100\*2÷40-16÷4=$$ | $$ 100\*2÷40-16÷4=$$$$=200÷40-4=$$$$=5-4=$$$$=1$$ |

**Expressions with Four Operations and Exponents**

Exponents are more powerful than multiplication or division.

If exponents are present in an expression, they are evaluated before any multiplication or division.

**Sample Problem 2**: **Find the value of each numerical expression.** **Follow the order of operations when finding each value.**

|  |  |  |
| --- | --- | --- |
| **a.** | $$56-12^{2}÷6+4=$$ | $$ 56-12^{2}÷6+4=$$$$=56-144÷6+4=$$$$=56-24+4=$$$$=32+4=$$$$=36$$ |
| **b.** | $$13^{2}\*10-15^{2}÷25=$$ | $$ 13^{2}\*10-15^{2}÷25=$$$$=169\*10-225÷25=$$$$=1,690-9=$$$$=1,681$$ |
| **c.** | $$125+216÷36-4^{2}\*6=$$ | $$ 125+216÷36-4^{2}\*6=$$$$=125+216÷36-16\*6=$$$$=125+6-96=$$$$=131-96=$$$$=35$$ |

**Expressions with Parentheses**The last important rule in the order of operations involves grouping symbols, usually parentheses.

These tell us that in certain circumstances or scenarios, we need to do things out of the usual order. Operations inside grouping symbols are always evaluated first, before exponents and any operations.

**Sample Problem 3**: **Find the value of each numerical expression.** **Follow the order of operations when finding each value.**

|  |  |  |
| --- | --- | --- |
| **a.** | $$6\*(12+112)=$$ | $$ 6\*(12+112)=$$$$=6\*124=$$$$=744$$ |
| **b.** | $$296-(65\*2-32÷4)=$$ | $$ 296-(65\*2-32÷4)=$$$$=296-(130-8)=$$$$=296-122=$$$$=174$$ |
| **c.** | $$2\*(625÷5\*2-40-28÷4)=$$ | $$ 2\*(625÷5\*2-40-28÷4)=$$$$=2\*(125\*2-40-7)=$$$$=2\*(250-40-7)=$$$$=2\*(210-7)=$$$$=2\*203=$$$$=406$$ |

**Expressions with Parentheses and Exponents**

**Sample Problem 4**: **Find the value of each numerical expression.** **Follow the order of operations when finding each value.**

|  |  |  |
| --- | --- | --- |
| **a.** | $$100-(8^{2}÷4\*2-24÷8)=$$ | $$ 100-(8^{2}÷4\*2-24÷8)=$$$$=100-(64÷4\*2-24÷8)=$$$$=100-(16\*2-3)=$$$$=100-\left(32-3\right)=$$$$=100-29=$$$$=71$$ |
| **b.** | $$20^{2}÷100+(7^{2}+10^{2}÷25)=$$ | $$ 20^{2}÷100+\left(7^{2}+10^{2}÷25\right)=$$$$=400÷100+\left(49+100÷25\right)=$$$$=4+\left(49+4\right)=$$$$=4+53=$$$$=57$$ |
| **c.** | $$550-\left(11^{2}-7^{2}\*2\right)^{2}=$$ | $$ 550-\left(11^{2}-7^{2}\*2\right)^{2}=$$$$=550-\left(121-49\*2\right)^{2}=$$$$=550-\left(121-98\right)^{2}=$$$$=550-\left(23\right)^{2}=$$$$=550-529=$$$$=21$$ |