Prime Factorization and Greatest Common Factor

Notes

Prime Factorization

Prime Numbers

A **Prime Number** can be divided evenly only by 1 or itself and it must be a whole number greater than 1. **Smallest Prime number is 2.** The first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29……

Factors

A **Factor** is a number that divides exactly into another number.

Prime Factorization

Factor a number is to break that number down into smaller parts. To find the prime factorization of a number, break that number down to its prime factors. "**Prime Factorization**" is finding which prime numbers multiply together to make the original number.

Some examples

**Example 1:** What are the prime factors of 14?
Always start from the smallest prime number, which is 2, so let's check:
14 ÷ 2 = 7
14 is divided evenly by 2. So, go to the next step
7 is a prime number
14 = 2 × 7
every factor is a prime number so, 2 and 7 are prime factors of 14.

**Example 2:** What are the prime factors of 18?
Always start from the smallest prime number, which is 2, so let's check:
18 ÷ 2 = 9
18 is divided evenly by 2. So, go to the next step
9 is not a prime number and 9 is not divided evenly by 2 so check with next prime number that is 3
9 ÷ 3 = 3
3 is a prime number
18 = 2 × 3 × 3
every factor is a prime number so, 2, 3 and 3 are prime factors of 18.

Another Method

Sometimes for bigger numbers it is easier to break a number down into any factors then break those factors down to primes.

**Example:** What are the prime factors of 80?
Break 80 into 8 × 10
Prime factors of 8 are 2 × 2 × 2
Prime factors of 10 are 2 and 5
Therefore, prime factors of 80 are **2, 2, 2 and 5**
Unique
There is only one (unique) set of prime factors for any number.
Example: The prime factors of 210 are 2, 3, 5 and 7:
\[ 210 = 2 \times 3 \times 5 \times 7 \]
There is no other possible set of prime numbers that can be multiplied to make 210.

Factor Tree

Given above is an example of factor tree, in which number is break down in its factors until we can't get any more factors and written in shape of tree. From this example we can see that prime factors of 42 is \(2 \times 3 \times 7\).

Greatest Common factor (GCF)

The greatest common factor is the largest of the common factors (of two or more numbers) or we can say the greatest number that divides exactly into two or more numbers. Greatest common factor (GCF) is also known as greatest common divisor (G.C.D) and highest common factor (HCF).

Finding the GCF using Prime factorization:

1. List the prime factors for each number.
2. Find the Prime Factors they have in common.
3. Multiply the factors common to both numbers and the answer is GCF.
4. If there are no common prime factors then the GCF is 1.

**Example1:** Find GCF of 54 and 96

Prime Factors of 54 are \(2 \times 3 \times 3 \times 3\)
Prime factors of 96 are \(2 \times 2 \times 2 \times 2 \times 2 \times 3\)
GCF = Common factor of both the numbers is 3. So, GCF of 54 and 96 is \(2 \times 3 = 6\).

**Example2:** Find GCF of 45 and 75

Prime Factors of 45 are \(3 \times 3 \times 5\)
Prime factors of 75 are \(3 \times 5 \times 5\)
GCF = Common factor of both the numbers is \(3 \times 5\). So, GCF of 45 and 75 is \(3 \times 5 = 15\).
Prime Factorization and Greatest Common Factor Guided Notes

Use the GCF and the Distributive Property to express the sum as a product

Step 1: Find the GCF of the 2 numbers
Step 2: Re-write using the distributive property.

Example
Use the GCF and the Distributive Property to express the sum $49 + 63$ as a product.
Prime factors of 49 are $7 \times 7$
Prime factors of 63 are $7 \times 9$
GCF of 49 and 63 is 7. Therefore, $49 + 63 \rightarrow 7(7 + 9)$

Exercise

1. Draw factor tree of the number 72.
2. Find G.C.F of 44 and 56?